

Number Series

1. How many terms are there in the series? 201, 208, 215, ----- 369

(A) 23

(B) 24

(C) 25

(D) 26

Answer: (C) 25

$$n = \frac{(l-a)}{d} + 1$$

where, l = last term = 369

$$n = \frac{(369-201)}{7} + 1$$

a = first term = 201

$$= \frac{(168)}{7} + 1 \Rightarrow 24 + 1 = 25$$

d = common difference

Ans : 25

2. $2^2 + 4^2 + 6^2 + \dots + 20^2 = \dots$

(A) 1155

(B) 1540

(C) 2310

(D) 385×385

Answer: (B) 1540

$$S_n = \frac{n(n+1)(2n+1)}{6}$$

$$= 2^2 (1^2 + 2^2 + 3^2 + 4^2 + \dots + 10^2)$$

Where ,

$$N=10 \quad = 2 \times 2 \left(\frac{10(10+1)(20+1)}{6} \right) = 2 \times 2 \left(\frac{2310}{6} \right)$$

$$= 2^2 (385) \Rightarrow 1540$$

3. Find the sum of the first 40 terms of the series $1^2 - 2^2 + 3^2 - 4^2 + \dots$

(A) 820

(B) – 820

(C) 870

(D) – 870

Answer: (B) – 820

If both sign were presented in the equation

$$-\frac{n(n+1)}{2} = -\frac{40(40+1)}{2} = -20(41) \\ = -820$$

4. Find the sum of the following series $2^2 + 3^2 + \dots + 20^2$

(A) 2867

(B) 2868

(C) 2869

(D) 2870

Answer: (C) 2869

$$\epsilon n2 = \frac{n(n+2)(2n+1)}{6} \\ \epsilon n2 = \frac{20(20+1)(2(20)+1)}{6} \\ \epsilon n2 = \frac{20(21)(41)}{6} \\ \epsilon n2 = 2870$$

But we should subtract 1^2 from the $\epsilon n2$ because the question starts from 2^2

$$2870 - 1 = 2869$$

5. $\frac{1}{1.2.3} + \frac{1}{2.3.4} + \frac{1}{3.4.5} + \frac{1}{4.5.6}$ is equal to

(A) $\frac{15}{31}$

(B) $\frac{7}{30}$

(C) $\frac{16}{21}$

(D) $\frac{21}{27}$

Answer: (B) $\frac{7}{30}$

$$= \frac{1}{6} + \frac{1}{24} + \frac{1}{60} + \frac{1}{120}$$

$$\text{Take LCM } \frac{20+5+2+1}{120} = 28/120$$

$$= \frac{7}{30}$$

6. Find the value of $(1 - 1/3) (1 - 1/4) (1 - 1/5) \dots (1 - 1/100)$

(A) $1 / 100$

(B) $1 / 50$

(C) $2 / 3$

(D) $99 / 100$

Answer: (B) $1 / 50$

$$6. \left(1 - \frac{1}{3}\right) \left(1 - \frac{1}{4}\right) \left(1 - \frac{1}{5}\right) \dots \dots \left(1 - \frac{1}{100}\right)$$

$$= \left(\frac{2}{3}\right) \left(\frac{3}{4}\right) \left(\frac{4}{5}\right) \dots \dots \left(\frac{99}{100}\right)$$

All the terms will be cancelled expect the first term 2 and last term 100

$$\frac{2}{100} = \frac{1}{50}$$

$$= \frac{1}{50}$$

7. In a geometric series, if the fourth term is $2 / 3$ and seventh term is $16 / 81$, then what is the first term of the series?

(A) $2 / 3$

(B) $4 / 9$

(C) $8/27$

(D) $9/4$

Answer: (D) $9/4$

7. 4th term = $\frac{2}{3} - \dots \rightarrow > 1$

7th term $\frac{16}{81} - \dots \rightarrow 2$

To find 1st term

$$Ar^3 = 2/3 \quad ar^6 = 16/81$$

$$a = \frac{2/3}{r^3} - \dots > 1$$

$$a = \frac{16/81}{r^6} - \dots > 2$$

$$\frac{2/3}{r^3} = \frac{16/81}{r^6} \cdot \frac{1}{r^6}$$

$$\Rightarrow r^3 = \frac{8}{7} = \frac{2}{3}$$

$$A(2/3)^{3/2} = (2/3)$$

$$A(2/3)^2 = 1$$

$$\frac{4a}{9} = 1$$

$$\Rightarrow a = \frac{9}{4}$$

8. The seventh term in the series 2, 6, 12, 20, 30,

(A) 42

(B) 72

(C) 56

(D) 90

Answer: (C) 56

Difference of each number increase by 2

Is 2+4, 6+6, 12+8, 20+10, 30+12

So the number is 42

9. The 20th term of 2, 10 30, 68, is

(A) 408

(B) 8020

(C) 820

(D) 420

Answer: (B) 8020

$$1^3 + 1 = 2, 2^3 + 2 = 10, 3^3 + 3 = 30$$

$$4^3 + 4 = 68$$

$$20^3 + 20 = 8000 + 20$$

$$= > 8020$$

10. Find the sum of first 20 multiples of 15

(A) 3150

(B) 3050

(C) 2750

(D) 2950

Answer: (A) 3150

$$\epsilon n = 15 + 30 + 45 + \dots \dots 300$$

$$= 15[1+2+3+4+5+\dots\dots+20]$$

$$\epsilon n = \frac{n(n+1)}{2} = 15 \left[\frac{20(20+1)}{2} \right]$$

$$= 15 (210)$$

$$= 3150$$

11. Find the sum of the first 20 terms of the series $1^2 - 2^2 + 3^2 - 4^2 + 5^2 - 6^2 + \dots$.

(A) - 420

(B) – 210

(C) 2870

(D) 420

Answer: (B) – 210

Both signs were present ,

$$\begin{aligned}\text{So } -\frac{n(n+1)}{2} &= -\frac{20(20+1)}{2} \\ &= -210\end{aligned}$$

12. $1 + \frac{1}{2} + \frac{1}{4} + \frac{1}{7} + \frac{1}{14} + \frac{1}{28}$ is equal to

(A) 2.5

(B) 2.0

(C) 1.5

(D) 1.0

Answer: (B) 2.0.

Take LCM = 28

$$\begin{aligned}&= \frac{28 + 14 + 7 + 4 + 2 + 1}{28} = \frac{56}{28} \\ &= 2\end{aligned}$$

13. If $1^3 + 2^3 + 3^3 + \dots + n^3 = 36100$ then $1 + 2 + 3 + \dots + n$ is equal to

(A) 290

(B) 190

(C) 390

(D) 490

Answer: (B) 190

$$1 + 2 + 3 + \dots + n = ?$$

$$\epsilon k3 = \epsilon k2$$

$$36100 = (1 + 2 + 3 + \dots)^2$$

$$1 + 2 + 3 + \dots n = \sqrt{36100}$$

$$N = 190$$

14. Find the sum of all-natural numbers between 300 and 500 which are divisible by 11.

(A) 7337

(B) 7227

(C) 7447

(D) 7557

Answer: (B) 7227

A = first number = 308

L = last number = 495

D = difference between each number = 11

$$\Rightarrow n = \frac{l-a}{d} = \left(\frac{495-308}{11} \right) + 1$$

$$N = 18$$

$$\Rightarrow \epsilon n = \frac{n}{2} (a + l)$$

$$= = \frac{18}{2} (308 + 495) = 9 (803)$$

$$\epsilon n = 7227$$

15. The product of n consecutive positive integers is divisible by

(A) $(n + 1)!$

(B) $n!$

(C) $(n - 1)!$

(D) $(n + 1) (n - 1)$

Answer: (B) $n!$

If $n = 1, 2, 3, 4$ then it will be divisible by n only

16. 5th term in series $\frac{2}{5}, \frac{6}{25}, \frac{18}{125}, \dots$

$$r = \frac{3}{5} (\text{difference})$$

$$a = \frac{2}{5} (\text{first number})$$

$$Ar^{n-1} = \frac{2}{5} \times 3^{5-1} / 5 = 2/5 \times 3^4 / 5$$

$$= \frac{81}{625} \times \frac{2}{5} = \frac{162}{3125}$$

(OR)

Take $\frac{2}{5}$ then $\frac{2}{5} (1, \frac{3}{5}, \frac{9}{25}, \frac{27}{125}, \frac{81}{625})$

Then take fifth term and multiply with $2/5$

$$\frac{81}{625} \times \frac{2}{5} = \frac{162}{3125}$$

17. The sum of all 3-digit numbers which are divisible by 8 is

(A) 61376

(B) 63176

(C) 67136

(D) 66137

Answer: (A) 61376

A = first number = 104

L = last number = 992

D = difference = 8

$$n = \left(\frac{l-a}{d} \right) + 1$$

$$n = \left(\frac{992-104}{8} \right) + 1 \quad n = 112$$

$$Sn = \frac{n}{2} (a + l)$$

$$= 56 (1096)$$

$$= 61376$$

18. The value of $\frac{1}{\sqrt{2}+1} + \frac{1}{\sqrt{3}+\sqrt{2}} + \frac{1}{\sqrt{4}+\sqrt{3}} + \dots + \frac{1}{\sqrt{9}+\sqrt{8}}$ is:

(A) $1 + \sqrt{2} + \sqrt{3} + \sqrt{4} + \dots + \sqrt{9}$

(B) $\sqrt{3} + \sqrt{5} + \sqrt{7}$

(C) $10\sqrt{5}$

(D) 2

Answer: (D) 2

$$18. \frac{1}{\sqrt{2}+1} + \frac{1}{\sqrt{3}+\sqrt{2}} + \frac{1}{\sqrt{4}+\sqrt{3}} + \dots + \frac{1}{\sqrt{9}+\sqrt{8}}$$

$$\begin{aligned} \text{If we conjugate 1 term} &= \frac{1}{\sqrt{2}+1} \times \frac{\sqrt{2}-1}{\sqrt{2}-1} = \frac{\sqrt{2}-1}{\sqrt{2}-1} \\ &= \sqrt{2} - 1 \end{aligned}$$

Then all other terms will be like this

$$\sqrt{2} - 1 + \sqrt{3} - 2 + \sqrt{4} - \sqrt{3} + \dots \dots \dots \sqrt{9} - \sqrt{8}$$

All the other terms expect $\sqrt{9}$ and -1

Will be other cancelled then $\sqrt{9} - 1 = 3 - 1 = > 2$

19. Find the sum of the first 20 terms of the geometric series $5/2 + 5/6 + 5/18 + \dots$

(A) $\frac{15}{4} \left[1 - \left(\frac{1}{3}\right)^{20} \right]$

(B) $\frac{15}{4} \left[1 - \left(\frac{1}{3}\right)^{18} \right]$

(C) $\frac{15}{4} \left[1 - \left(\frac{1}{3}\right)^{16} \right]$

(D) $\frac{15}{4} \left[1 - \left(\frac{1}{3}\right)^{14} \right]$

Answer: (A) $\frac{15}{4} \left[1 - \left(\frac{1}{3}\right)^{20} \right]$

19 . Sum of first 20 terms of geometric series $\frac{5}{2} + \frac{5}{6} + \frac{5}{18} + \dots$

A = first term = $\frac{5}{2}$ n = total no. of terms

R = difference = $\frac{1}{3}$

$$\sum_{k=0}^n ((a^k)) = 1 \left(\frac{1 - rn}{1 - r} \right)$$

$$= \frac{5}{2} \left(\frac{1 - \left(\frac{1}{3}\right)n}{1 - \frac{1}{3}} \right)^{20}$$

$$= \frac{5}{2} \left(\frac{1 - \left(\frac{1}{3}\right)}{\frac{3-1}{3}} \right)^{20}$$

$$= \frac{5}{2} \left(\frac{1 - \left(\frac{1}{3}\right)}{\frac{2}{3}} \right)^{20}$$

$$\Rightarrow \frac{15}{4} \left[1 - \left(\frac{1}{20}\right)^{20} \right]$$